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# UNCERTAINTY OBSERVER-BASED I/O LINEARIZING CONTROL FOR THE REGULATION OF A CONTINUOUS WASTEWATER BIOREACTOR FOR CD REMOVAL*

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## ABSTRACT

In this work an uncertainty observer based controller in order to regulate a class of highly nonlinear system is considered. It is assumed that the main nonlinearities and the input disturbances of the system are unknown. In order to avoid the above drawback, an uncertainty observer, which is based on a class of high-order polynomial output injection, to infer the unknown terms was proposed. The above mentioned uncertainty observer is coupled with an Input-Output (I/O) linearizing controller in order to design a robust methodology against model uncertainties and input disturbances. The proposed control scheme is then applied to a continuous sulfate-reducing bioreactor with application to heavy metal (Cadmium) removal. The proposed kinetic model of the bioreactor was experimentally corroborated and delivered satisfactory correlation coefficients in comparison with the selected experimental data. Numerical simulations show the satisfactory performance of the proposed methodology in comparison with a standard linear controller.

**Keywords**: bioprocess, variable structure control, observability, wastewater treatment

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## INTRODUCTION

The control of uncertain nonlinear systems is a key and challenging problem in process operation, as commonly, the behavior of industrial processes is extremely nonlinear. Numerous high quality publications related with new nonlinear controller design have been proposed in the last decade, where robust design, variable structure system (VSS) control, optimal control, neural networks (NNs), etc. have an important mention (Isidori, 1995; Gouzé et al., 2000; Nam and Jin 2000; Chen and Yang, 2007; Frikha et al., 2007; Liao and Yu, 2008; Magni et al., 2009; Chen and Chen, 2010). In general, robust control of nonlinear systems is usually achieved using one of two schemes. The first scheme involves the design of a controller that is insensitive, as possible, to model uncertainty and/or disturbance inputs. The second scheme involves adapting model parameters or control gains in real-time in order to achieve the desired performance. Adaptive control methods fall into this category (Krstic, 1995; Jeon, and Choi, 2002; Jain et al., 2005). These control schemes can be used to provide robustness in a dynamic system with uncertainties, each with its own advantages and disadvantages (Tong and Li, 2010). Uncertainty in a dynamic system may take many forms, but among the most significant are noise/disturbance uncertainty and modeling/parametric uncertainty. The current design of robust controllers synthesizes a control law that maintains system responses and error signals to within pre-specified tolerances despite the effects of uncertainty on the system. Several robust control techniques have been applied to linear systems, such as $H\infty$ controllers (Skogestad and Postlethwaite, 2005; Wang, et al., 2009). These are based on singular value decompositions using weighted functions at varying frequencies for closed-loop specifications, which include sensitivity (disturbance), complementary sensitivity (tracking), and control input requirements. Singular values can also be used to provide information in terms of guaranteed bounds on system performance or tolerable perturbations.

The control of nonlinear systems is inherently more difficult than of linear systems. Variable structure control (VSC) under sliding-mode framework is an evolving method for robust control of nonlinear systems that provides stability in the presence of modeling uncertainties (Utkin, 1993; Perruquetti and Barbot, 2005). Variable structure control requires that the bounds on the modeling uncertainties are known in order to provide robust stability. Aside from that chattering in the control signal, which is highly undesirable in many systems, can occur because of the conservative nature of the controller. Chattering can be eliminated by smoothing the control signal within a thin boundary layer, however that comes with the cost of a slight reduction in the controller's performance over the system. Under this framework the so-called high order sliding-mode controllers have been proposed in order to provide smoothness to the control input and to avoid chattering (Li et al., 2003; Aguilar-López et al., 2009; Tang and Cai, 2011).

In this work a novel uncertainty observer, constructed via a class of polynomial output injection coupled with a linearizing input-output controller is presented for its application over a continuous sulfate-reducing bioreactor for heavy metal (Cadmium) removal. The main goal of the contolller is to enhance the system's cadmium removal by regulating the sulfide concentration within the fermenter, which is the most easily measured output of the process. Furthermore, it is proved that the closed-loop system is stable. Simulation analysis of the chosen continuous bioreactor using a kinetic model experimentally corroborated is employed as a benchmark to illustrate the performance of the proposed methodology.

## WASTEWATER TREATMENT

The indiscriminate use of natural resources is regularly accompanied by an increase in levels of local and global pollution, which is reflected with an imbalance of the ecosystem. Human activity and population growth are deemed as the main cause of environmental issues. The generation of large quantities of wastewater with high organic and toxic content is an obvious product of excessive consumption (Ziauddin et al., 2013). Water quality standards are increasingly stringent, pollutants such as pharmaceuticals, endocrine disruptors, pesticides and heavy metals have brought new scrutiny to water treatment and distribution systems established in developed countries (Qu et al., 2013).

Biological treatments are an important and integral part of any municipal or industrial wastewater treatment plant with soluble organic impurities or a mixture of the two types of wastewater (Tantak et al., 2014). The main objective of conventional biological treatment of wastewater is to reduce the load of dissolved organic carbon, phosphorus and nitrogen to prevent oxygen depletion and eutrophication of the receiving waters. Therefore, negative environmental impacts have fueled our need to understand the effect of pollution on water bodies and appropriate measures to reduce it, including the potential use of microorganisms in treatment processes. (Völker et al., 2016; Ziauddin et al., 2013). Some studies focus mainly on the potential of microorganisms (such as bacteria and fungi) in the removal of heavy metals. These biological treatment technologies may be coupled with physicochemical processes to enhance its efficiency and reduce their costs.

## IMPORTANCE OF WASTEWATER TREATMENT WITH HEAVY METALS

Heavy metals are generally considered to be those whose density is greater than 5 g per cubic centimeter and are an important group among the pollutants present in wastewater, as they're associated with the occurrence of serious health effects such as reduced growth and development, cancer, damage to the central nervous system, development of autoimmunity, and in extreme cases death. Electroplating, metal surface treatment, printed circuit board manufacturing, wood processing industry, inorganic pigment manufacturing, petroleum refining and photographic operations generate significant amounts of wastewater containing heavy metals (cadmium, Zinc, lead, chromium, nickel, copper, vanadium, platinum, silver, titanium, arsenic), (Barakat, 2011).

Among the heavy metals cadmium is one of the most important environmental pollutants. It is used in the manufacture of food containers, alloys, paints, batteries, leather, etc. Although it is one of the most toxic metals, it is an essential element necessary in micro quantities for bacterial growth, but its role into the metabolism of some species is still not clear. The NORMA OFICIAL MEXICANA (NOM-001-ECOL-1996), indicates that the maximum permissible cadmium limit is 0.05 mg/L in soil and 0.1 mg/L in rivers and it has been classified as a priority contaminant by the US Environmental Protection Agency (EPA), (Igwe and Aiba, 2006; López Pérez et al., 2015).

Microorganisms have the ability to remove a large variety of organic pollutants (Das et al., 2008), however it is worth mentioning that microorganisms generally do not have the capacity to mineralize metals, but some species can modify their oxidation state to a less toxic form either by the excretion of a metabolite or via enzymatic action. The most serious form of cadmium toxicity in humans is called "itai-itai," a disease characterized by unbearable pain in the bones. Other implications for the health of cadmium in humans include renal dysfunction, liver damage and hypertension. For this reason, the disposal of this pollutant from the environment, particularly wastewater, has received major attention in the last decade (López Pérez et al., 2015).

## Sulfate Reducing Bacteria (SRB) in Wastewater Treatment

Microorganisms are capable of removing heavy metals from wastewater by three known mechanisms: Precipitation, accumulation and absorption. An example of the first one are sulfate reducing bacteria (SRB), which through the production of hydrogen sulfide are capable of precipitating metals. SRB form a group of prokaryotes capable of transforming sulfate into hydrogen sulfide ($H_2S$). SRB members are widespread in anoxic habitats, where they play an essential role in both carbon and sulfur biogeochemical cycles (Castro et al., 2000; López Pérez et al., 2015)

Under anaerobic conditions, SRB oxidizes simple organic compounds using sulfate as an electron acceptor, where sulfate is reduced to sulfur (Muyzer and Stams, 2008; Postgate 1981). The generation of sulfide produces reducing conditions, elimination of acidity and precipitation of metals from the solution as sulfides. This property makes these bacteria suitable for eliminating acidity and metals from contaminated effluents (Muyzer and Stams, 2008). The elimination method consists of two stages: (1) the biogenic $H_2S$ production generated by the SRB during anaerobic respiration and (2) the reaction of biogenic $H_2S$ with metal ions, which produces insoluble metal sulphides that can be easily separated from an aqueous solution (Jong and Parry, 2003). In addition, SRB are capable of forming extracellular polymeric substances (EPS) as a protective mechanism towards heavy metal exposure, such compounds are generally made of a complex mixture of polysaccharides, proteins, nucleic acids and phospholipids (Flemming and Wingender, 2001).

EPS can bind to significant amounts of potentially toxic metals, therefore they're considered as great material for biosorption processes. In this regard, bacteria of the genus *Desulfovibrio* are one of the most studied in this field, showing a high efficiency in the elimination of different metal ions in the range of a few milligrams per liter up to 100 mg/L, for example Zn (25-40 mg/L) , Ni (10-20 mg/L), Pb (75-80 mg/L), Cu (4-20 mg/L), Cd (>4-20 mg/L) (Cabrera et al., 2006; López Pérez et al., 2015; Utgikar et al., 2002).

## PROCESS DESCRIPTION

The search for efficient and cheap technologies that would allow for the elimination or recycling of sulfate and metals present in these effluents has become a necessity. Sulfate reduction relying on the activity of sulfate–reducing bacteria (SRB) is of interest for its potential for the simultaneous removal of sulfate and heavy metals (López et al., 2012). Numerous studies concluded that this microbial sulfate reduction can decrease sulfate and metal concentrations in aqueous streams to very low levels suitable for environmental discharge (Bhagat et al., 2004; Peña-Caballero et al., 2012).

In order to improve process understanding or performance different tools can be considered as simulators able to reproduce system behavior such as software sensors, virtual sensors and observer states, etc. which allow obtaining an estimation of unmeasured signals based on the monitoring of easily measured ones. All these tools rely on a representation of the considered system, which is generally a mathematical model of the process. Dochain (2003) developed a methodological approach for modeling batch bioreactors, however unstructured kinetic models are frequently employed for the characterization of bacterial growth, substrate consumption and product formation in bioprocesses. Currently both Gompertz and Ludeking-Piret type structures have been used to describe the dynamics of product generation or substrate degradation in biological systems (López and Borzacconi, 2010), however these structures consider the process just as a dependence of either time of biomass growth respectively, while in reality much of the bioremediation reactions made by *Desulfovibrio* bacteria are generally decoupled from the primary metabolism of the culture making them inaccurate and thus, unsuitable for process control strategies. Novel developed mathematical models that consider more biochemical and genetic information are being developed to overcome such troubles, but generally these are complex enough to make them unsolvable analytically.

For this reason, novel mathematical model development is needed to describe the kinetics of both the bacterial growth and the bioremediation for wastewater treatment in batch systems (Dilution rate, $D = 0$) and also for further analysis, considering a continuous operation for control purposes must be considered (Dilution rate, $D > 0$). The prior assumption is valid as long as the reactor can be operated near perfect mixing conditions, which can be scaled up reliability at least up to pilot level as validated experimentally for similarly modelled systems (Ariyajaroenwong et al., 2016). Therefore, for the system representation of the sulphate-reducing reactor for Cd removal from wastewater a homogeneous Non-Structured mathematical model previously reported by López-Pérez et al., (2012) was chosen:

Lactate mass balance (L):

$$\frac{d\delta_L}{dt} = D\left(\delta_{L\,in} - \delta_L\right) - k_{LA} Y_{L/X} \left[\frac{k_{ace}}{\delta_A + k_{ace}}\right]\left[\frac{\delta_L{}^{\theta}}{k_{lac} + \delta_L{}^{\theta}}\right]\delta_X \tag{1}$$

Acetate mass balance (A):

$$\frac{d\delta_A}{dt} = -D\delta_A + k_{LA} Y_{A/X} \left[\frac{k_{ace}}{\delta_A + k_{ace}}\right]\left[\frac{\delta_L{}^{\theta}}{k_{lac} + \delta_L{}^{\theta}}\right]\delta_X \tag{2}$$

Sulfate mass balance ( $\delta_{SO_4^{-2}}$ ):

$$\frac{d\delta_{SO_4^{-2}}}{dt} = D\left(\delta_{SO_4^{-2}\,in} - \delta_{SO_4^{-2}}\right) - \frac{k_{spx}}{Y}\left(1 - \frac{\delta_{H_2S}}{k_p}\right)^{\alpha}\left[\frac{\delta_{SO_4^{-2}}}{k_s + \delta_{SO_4^{-2}}}\right]\delta_X\ \delta_L{}^{\varepsilon} \tag{3}$$

Sulfide mass balance ( $\delta_{H_2S}$ ):

$$\frac{d\delta_{H_2S}}{dt} = -D\delta_{H_2S} + \frac{k_{spx}}{Y_p}\left(1 - \frac{\delta_{H_2S}}{k_p}\right)^{\alpha}\left[\frac{\delta_{SO_4^{-2}}}{k_s + \delta_{SO_4^{-2}}}\right]\delta_X\ \delta_L{}^{\varepsilon} \tag{4}$$

Biomass Balance ( $\delta_X$ ):

$$\frac{d\delta_X}{dt} = -D\delta_X + k_{spx}\left(1 - \frac{\delta_{H_2S}}{k_p}\right)^{\alpha}\left[\frac{\delta_{SO_4^{-2}}}{k_s + \delta_{SO_4^{-2}}}\right]\left(1 - \frac{\delta_{Cdl}}{k_{Cdl}}\right)^{\beta}\delta_X\ \delta_L{}^{\rho} - k_d\ \delta_X\ \delta_L{}^{\rho} \tag{5}$$

Biofilm mass balance ( $\delta_B$ ):

$$\frac{d\delta_B}{dt} = -D\delta_B + k_{bio}\left(1 - \frac{\delta_B}{k_B}\right)^{\beta}\delta_X\ \delta_L \tag{6}$$

Cadmium in liquid mass balance ( $\delta_{Cdl}$ ):

$$\frac{d\delta_{Cdl}}{dt} = D(\delta_{Cdl\,in} - \delta_{Cdl}) - k_{Cdl}\left(1 - \frac{\delta_{Cdb}}{k_{spx}}\right)^{\chi}\left[\frac{\delta_{Cdl}}{k_1 + \delta_{Cdl} + \frac{\delta_{Cdl}^{\ 2}}{k_2}}\right]\delta_X\ \delta_B \tag{7}$$

Cadmium in biofilm mass balance ( $\delta_{Cdb}$ ):

$$\frac{d\delta_{Cdb}}{dt} = D(\delta_{Cdb}) + k_{Cdl}\left(1 - \frac{\delta_{Cdb}}{k_{spx}}\right)^{\chi}\left[\frac{\delta_{Cdl}}{k_1 + \delta_{Cdl} + \frac{\delta_{Cdl}^{\ 2}}{k_2}}\right]\delta_X\ \delta_B \tag{8}$$

## PROBLEM STATEMENT

A bioprocess in a stirred tank bioreactor can be described by a set of $k$ coupled microbiological and biochemical reactions which take place in the reactor and involve a set of $n$ biological or chemical species such as microorganisms, substrates, metabolites, enzymes, etc. The dynamical behavior of a continuously stirred tank bioreactor (CSTR) is often described by a general mass balance model in which the trajectories of the system are (lower and upper) bounded on $\Omega$ for any given bounded inputs and given positive initial conditions.

Let us consider the following specific nonlinear representation of such a class of nonlinear system, with linear measured output:

$$\dot{x} = Ax + \Psi(x) + (g_0 + \Delta g)u + j(x)d \tag{9}$$

$$y = h(x) = Cx \tag{10}$$

where $x \in \Re^n$ is the state vector, $\Psi(x) := \Re^n \to \Re^n$ is a nonlinear smooth vector function, $u \in \Re^q; q \le n$ is the control input, $j(x) := \Re^n \to \Re^m, m \le n, g_0$ is the nominal control input coefficient, $\Delta g$ is the bounded additive uncertainty of the control input and $d \in \Re^l; l \le n$ is the bounded input disturbance vector.

Now, considering that $\Psi(x), \Delta g$ and $h(x)d$ are unknown, the following change of variable is proposed:

$$w = \Psi(x) + \Delta g u + h(x) d \tag{11}$$

Therefore the following uncertain extended system (Aguilar, et al., 1997, 2001) is considered:

$$\dot{x} = Ax + g_0 u + w \tag{12a}$$

$$\dot{w} = \wp(x, \bar{u}) \tag{12b}$$

where $\bar{u} = (u, d)$, $\wp := \Re^{n+q+z} \rightarrow \Re^n$ is an unknown vector field which is assumed that satisfies a Lipschitz condition, respect to the vector $x$, i. e.,

$$\left\| w(x, \bar{u}) - \hat{w}(\hat{x}, \bar{u}) \right\| \leq L \left\| x - \hat{x} \right\| \tag{13}$$

and considering that $|\wp| \leq \Omega < \infty$.

## Uncertainty Estimator Based Observer Design

Because of the increasing complexity and necessity for safety of industrial processes, efficient monitoring and decision support systems are becoming more and more important. Indeed, even in normal operational conditions, several types of disturbances may occur with serious consequences in the performance of the process. Hence, there is a clear need for advanced control in order to keep the system performance as close as possible to optimal.

As showcased into the prior section of this work, biotechnological processes in stirred tank reactors can be modeled by nonlinear differential equations involving nonlinear biological kinetics (e.g., cellular growth rate, bioproduction rate, etc.). Very often, the precise formulation of these kinetics is not well known, and rough approximations and hypotheses are made to write the model.

Particularly in wastewater treatment the following factors of uncertainty are true, and as such they clearly show the need for monitoring systems and automatic control in order to optimize the process operation or to detect disturbances:

- Parametric uncertainty: A great number of bacterial species carry out the transformations of organic load and nutrients in wastewater treatment processes without direct or easily comprehensible relationships between the microbial populations and viability, and key aspects such as cellular physiology and its modeling are not easily understood from external measurements (Dochain and

Perrier, 1998; van Veldhuizen, van Loosdrecht, and Heijnen, 2001). As a first consequence, the kinetics of these transformations is often poorly or inadequately known (van Impe and Bastin, 1998).

- Nonlinearity: In addition, it is well known that the process kinetics shows a highly nonlinear behavior. This a serious drawback in instrumentation and automatic control because, in contrast to linear systems where the observability can be established independently of the process inputs, the nonlinear systems must accomplish with the detectability condition depending on the available on-line measurements, including process inputs in the case of non-autonomous systems (Gauthier, Hammouri, and Othman, 1992).
- Lack of on-line sensors: A third factor of uncertainty arises from the lack of adequate sensors to measure on-line all the important variables in the process. Even when some of these sensors are available commercially nowadays, they are still expensive, time consuming and require additional expenses for the installation and use on-site (Huntington, 1998).

In an ideal scenario let us now suppose that we are able to measure without noise some outputs, and that we want to build an observer to estimate the state variables. Some rather general methodologies exist for nonlinear system and in particular for biological ones, such as the well-known classical extended Kalman filters (EKF) and Luenberger observers (ELO) which allow the estimation of both the parameters and the state of the system. One of the reasons for the popularity of these estimators is that they are easy to implement since the algorithm can be derived directly from the state space model. However, since these estimators are based on a linearized model of the process, the stability and convergence properties are essentially local and valid only around an operating point and it is rather difficult to guarantee its stability over wide ranges of operation; those shortcomings coupled with the fact of the generally bad quality of the models used in biotechnology, it is often preferable to build a different kind of observer (often called asymptotic observer), not using the unknown parts (biological kinetics) of the model. In the counterpart these observers have weaker properties concerning, for example, the rate of convergence (Bastin and Dochain, 1990). These observers are based on mass balance considerations, and since they are in open-loop, they are not very robust to errors on the estimation of the influent masses.

Asymptotic observers are based on mass balance considerations, but in some cases the exact mass flow rate entering in the system is not well known. This is for example the case for wastewater treatment when the exact mass of polluting substrate in the influent is not measured. In that case of corrupted information on the system inputs, the predictions of the mass balance will be biased. It is worth noting that, due to their specific constructions, these observers are in open loop. Indeed the predictions of these observers are not compared with any measurements in order to correct the estimations. In the following we will show that such a correction is possible when an online measurement is a function of the state variables.

As above mentioned, for control purposes, it is needed an estimation of the uncertain term $w$ in order to made the considered control realizable. Therefore the following uncertainty observer is considered, (Aguilar-Sierra et al., 2010; Mata-Machuca, et al., 2011).

Proposition 1: The following dynamical system is an asymptotic observer for the system (12):

$$\dot{\hat{x}} = A\hat{x} + g_0 u + \hat{w} + \sum_{i=1}^{m} K_i (y - C\hat{x})^{2i-1} \tag{14}$$

$$\dot{\hat{w}} = \sum_{i=1}^{m} \overline{K}_i (y - C\hat{x})^{2i-1} \tag{15}$$

Considering the following assumptions:

- $w$ is observable with respect to {u, y}, that is to say, w satisfies a differential polynomial P in terms of {u, y} and some of their time derivatives; $P(w, u, \dot{u}, \ldots, y, \dot{y}, \ldots) = 0$
- K1 is selected from the following Ricatti algebraic equation, which has a symmetric and positive definite solution P for some $\rho > 0$.

$$(A - K_1 C)^T P + P(A - K_1 C) + L^2 PP + I + \rho I = 0 \tag{16}$$

- Ki is selected such that $\lambda_{\min}(PK_i C) \geq 0$, $2 \leq i \leq m$

  Defining the estimation error as:

$$\xi^T = (\xi_x, \xi_w)$$

where:

$$\xi_x = x - \hat{x} \tag{17}$$

$$\xi_w = w - \hat{w} \tag{18}$$

Therefore the corresponding dynamic equation of the estimation error is:

$$\dot{\xi}_x = (A - K_1 C)\xi_X - \sum_{i=2}^{m} K_i (C\xi_X)^{2i-1} + w - \hat{w} \tag{19}$$

$$\dot{\xi}_w = \wp - K_1\left(y - C\hat{x}\right) - \sum_{i=2}^{m} K_i\left(y - C\hat{x}\right)^{2i-1} \tag{20}$$

Sketch of proof of Proposition 1

Now, let us to consider the following Lyapunov function candidates:

$$\begin{aligned} \Im &= \Im_1 + \Im_2 \\ \Im_1 &= {\xi_x}^T P \xi_x \end{aligned} \tag{21}$$

$$\Im_2 = \tfrac{1}{2}{\xi_w}^2 \tag{22}$$

where $0 < P = P^T$

Now, $\dot{\Im}_1 = {\dot{\xi}_x}^T P \xi_x + {\xi_x}^T P \dot{\xi}_x$

$$\dot{\Im}_1 = {\xi_x}^T\left[\left(A - K_1C\right)^T P + P\left(A - K_1C\right)\right]\xi^T + 2{\xi_x}^T P\left[w - \hat{w}\right] - 2\sum_{i=2}^{m}\left(C\xi_x\right)^{2i-1} PK_1C\xi_x \tag{23}$$

Considering the Lipschitz condition we have:

$$2{\xi_x}^T P\left[w - \hat{w}\right] \le L^2 {\xi_x}^T PP\xi_x + {\xi_x}^T \xi_x \tag{24}$$

Applying the Rayleigh inequality and considering $\lambda_{min}\left(PK_iC\right) \ge 0$

$$-{\xi_x}^T PK_iC\xi_x \le -\lambda_{min}\left(PK_iC\right)\left\|\xi_x\right\|^2 \tag{25}$$

Therefore, maximizing the above equation:

$$\dot{\Im}_1 \le \xi\left[\left(A - K_1C\right)^T P + P\left(A - K_1C\right) + L^2PP + I\right]\xi^T - 2\sum_{i=2}^{m}\left(C\xi_x\right)^{2i-2}\lambda_{\min}\left(PK_iC\right)\left\|\xi_x\right\|^2 \tag{26}$$

$$\dot{\Im}_1 \le -\rho\left\|\xi_x\right\|^2 - 2\sum_{i=2}^{m}\left(C\xi_x\right)^{2i-2}\lambda_{\min}\left(PK_iC\right)\left\|\xi_x\right\|^2 \le 0 \tag{27}$$

$$\dot{\Im}_1 \le -\left(\rho + 2\sum_{i=2}^{m}\left(C\xi_x\right)^{2i-2}\lambda_{\min}\left(PK_iC\right)\right)\left\|\xi_x\right\|^2 \le 0 \tag{28}$$

Taking in account that:

$$\left(\rho + 2\sum_{i=2}^{m} \left(C\xi_x\right)^{2i-2} \lambda_{\min}\left(PK_iC\right)\right) > 0 \tag{29}$$

Now:

$$\dot{\Im}_2 = \xi_w \dot{\xi}_w = \xi_w \left( \wp - K_1\left(y - C\hat{x}\right) - \sum_{i=2}^{m} K_i\left(y - C\hat{x}\right)^{2i-1} \right) \tag{30}$$

$$\dot{\Im}_2 = \wp\,\xi_w - K_1 C\xi^2{}_w - \sum_{i=2}^{m} K_i\left(C\xi_w\right)^{2i} \tag{31}$$

Maximizing the above equation, the following holds:

$$\dot{\Im}_2 \le \Omega\left\|\xi_w\right\| - K_1C\left\|\xi_w\right\|^2 - \sum_{i=2}^{m} K_i\left(C\xi_w\right)^{2i} \tag{32}$$

$$\dot{\Im}_2 \le -\left(K_1C\left\|\xi_w\right\| - \Omega\right)\left\|\xi_w\right\| - \sum_{i=2}^{m} K_i\left(C\xi_w\right)^{2i} \le 0 \tag{33}$$

Considering that: $\sum_{i=2}^{m} K_i\left(C\xi_w\right)^{2i} > 0$

Besides, K1 can be selected to generate $K_1C\left\|\xi_w\right\| - \Omega > 0$.

Therefore:

$$\dot{\Im}_2 \le 0 \tag{34}$$

Note that $\dot{\Im}_2$ is negative on the set $\left\{ \left\|\xi_w\right\| \le \lim_{t\to\infty} \sup \frac{\Omega}{K_1C} \right\}$.

From the above, can be concluded that:

$$\dot{\Im} \le 0 \tag{35}$$

## ROBUST LINEARIZING CONTROLLER DESIGN

Apart from the traditional Luenberger derived observers, alternative methods for the design of nonlinear asymptiotic observers have been examined. Walcott and Zak (1987) investigated an observer design technique utilizing theory of variable-structure systems (VSS) and Slotine et al., (1987) discussed the potential use of sliding surfaces for observer design. Tsinias (1989) provided a sufficient Lyapunov-like condition for the existence of a nonlinear observer and showed that it is equivalent to detectability condition for linear case. However, in general, the construction of this Lyapunov function is quite difficult. Gauthier et al., (1992) showed that if a nonlinear system is uniformly observable for any inputs and some functions are globally Lipschitz, then there exists a nonlinear observer whose gain depends on the solution of some Lyapunov-like equation. Ciccarella et al., (1993) proposed a nice extension of the Luengerger-like observer for nonlinear systems of full relative degree under the global Hölder condition for certain functions. However, if nonlinear system has relative degree less than system order, their technique requires additional assumption that some time derivatives of the input should be zero almost everywhere. Therefore techniques that propose the use of a global nonlinear observer that guarantees the estimation error to converge to zero asymptotically are now based on the input output linearization technique. In contrast to Ciccarella et al., (1993), said proposition do not require the hypothesis of full relative degree, as its main assumption is concerned with the existence of nonlinear observer for internal dynamics.

Thus, the proposed technique can be regarded as a dual of stabilization problem via input–output linearization, since the latter is solvable if the zero dynamics of nonlinear system have a globally asymptotically stable equilibrium at the origin (Byrnes and Isidori, 1991). Moreover, as far as the local observation problem is concerned, the proposed condition is reduced to that the zero dynamics have a locally exponentially stable equilibrium at the origin.

The technique of state feedback linearization (Hunt, Su and Meyer, 1983) was developed in the effort of designing an autopilot for helicopters (Meyer, Su and Hunt, 1984). It requires measurements of the state vector $x$ and knowledge of the parameter vector $p$ in order to transform a multi-input nonlinear control system $(x \in R^n, u \in R^m, p \in R^q)$ into a linear and controllable one $(z \in R^n, v \in R^m)$

$$\dot{z} = Az + Bv \tag{36}$$

Let us consider again the equation (12.a):

$$\dot{x} = Ax + g_0 u + w$$

Considering the term $w$ as uncertain, the corresponding no ideal linearizing I/O controller can be proposed as:

$$u = g_0^{-1}\left(-g_1\left(x - x_{sp}\right) - Ax - \hat{w}\right) \tag{37}$$

where the estimate of the uncertain term $\hat{w}$ is provided by the uncertainty estimator given by equations (12-13), under this framework an observer based controller is performed.

Now, defining the control error as $\xi = x - x_{sp}$ where $x_{sp}$ is the required trajectory or set point, therefore the control error dynamic under the control law (27) is determinate by:

$$\frac{d\xi}{dt} = g_1\xi + \left(w - \hat{w}\right) \tag{38}$$

The above, assuming a regulation case, where $x_{sp} = cte$ and $\dot{x}_{sp} = 0$

or

$$\frac{d\xi}{dt} = g_1\xi + \xi_\omega \tag{39}$$

Now, for the above considerations the following assumptions are satisfied. There exist $g_1 \in \Re$ such that:

A1- $\left\|\xi_\omega\right\| \leq \dfrac{\Omega}{K_1 C}$

A2- $\lim\limits_{t \to t_0}\left\|\exp\left(-\int_0^t g_1\, d\sigma\right)\right\| = 0$,

where $t_0$ is large enough.

Equation (38) can be solved to obtain the error dynamics in the time domain as follows;

$$\xi = \xi_0 \exp\left(-\int g_1\, dt\right) + \int_0^t \exp\left(-g_1(t-\sigma)\right)\xi_\omega\, d\sigma \tag{40}$$

Taking norms of both sides of Equation (30) yields to an inequality, which is limited after assumptions A1 and A2.

$$0 \le \limsup_{t\to t_0} \|\xi\| \le \|\xi_0\| \limsup_{t\to t_0} \left[\left\|\exp\left(-\int g_1\,dt\right)\right\|\right] + \frac{\limsup_{t\to t_0}\left[\int_0^t \left\|\exp\left(\int g_1\,dt\right)\xi_\omega\,d\sigma\right\|\right]}{\limsup_{t\to t_0}\left\|\exp\left(\int g_1\,dt\right)\right\|} \tag{41}$$

$$0 \le \limsup_{t\to t_0} \|\xi(t)\| \le \frac{\limsup_{t\to t_0}\left[\frac{\Omega}{K_1 C}\int_0^t \left\|\exp\int g_1\,d\sigma\right\|\right]}{\limsup_{t\to t_0}\left\|\exp\left(\int g_1\,dt\right)\right\|} \tag{42}$$

Equation (42) is a case of an undefined quotient where L'Hôpital's rule, which can be applied to solve the corresponding equation. By considering the limit when $t \to t_0$ it is possible to obtain a bound for the estimation error:

$$0 \le \limsup_{t\to t_0} \|\xi(t)\| \le \limsup_{t\to t_0} \frac{\frac{\Omega}{K_1 C}\left\|\exp\left(\int g_1\,dt\right)\right\|}{\left\|\exp\left(\int g_1\,dt\right)\right\|\|g_1\|} = \limsup_{t\to t_0} \frac{\Omega}{\|g_1\| K_1 C} \tag{43}$$

Note that the control error can be diminished increasing the corresponding control´s gain *g1* however a large control action can made unstable the closed-loop behavior of the process via the exciting of non modeled dynamics and output disturbances; alternatively the control error can be diminished via the action of the proposed observer although the corresponding observer gain *K1*.

## NUMERICAL RESULTS

Several numerical simulations where done in order to show the performance of the proposed methodology, the system equations were solve employing ode23s library from Matlab™, the initial conditions for the bioreactor model are $S_0$ = 6000 mg/L; $P_0$ = 15 mg/L, $X_0$: = 50 mg/L, $L_0$ = 4250 mg/L, $A_0$ = 5 mg/L, $B_0$ = 0.5 mg/L, $Cdb_0$ = 0, $Cdl_0$ = 170 mg/L for sulfate, sulfide, biomass, lactate, acetate, biofilm and cadmium concentrations respectively. The control´s objective is to regulate the sulfide concentration in the bioreactor considering the dilution rate (input flow) as a control input proposing the sulfide concentration as the measured output of the bioreactor, which is a realistic assumption; the adequate sulfide concentration regulation provokes the indirect cadmium removal via Cadmium sulfide precipitation (Figure 1).

For comparison purposes, a well tuned PI controller is implemented, considering IMC tuning rules (Rivera et al., 1986), where the steady state gain is evaluated as $K$ = 1, the proportional gain is $k_p$ = 1.5 and the integral time is $\tau_I$ = 10.5 h .

The proposed uncertainty observer considers the sulfide kinetic as the unknown modeling term. The initial conditions of the proposed observer are ($S_0$ = 5000 mg/L; $P_0$ = 15 mg/L, $X_0$: = 50 mg/L, $L_0$ = 4000 mg/L, $A_0$ = 15 mg/L, $B_0$ = 0.05 mg/L, $Cdb_0$ = 0 mg/L, $Cdl_0$ = 170 mg/L) and the controlller gain of the linearizing controller is selected as $g_1$ = 2.5.

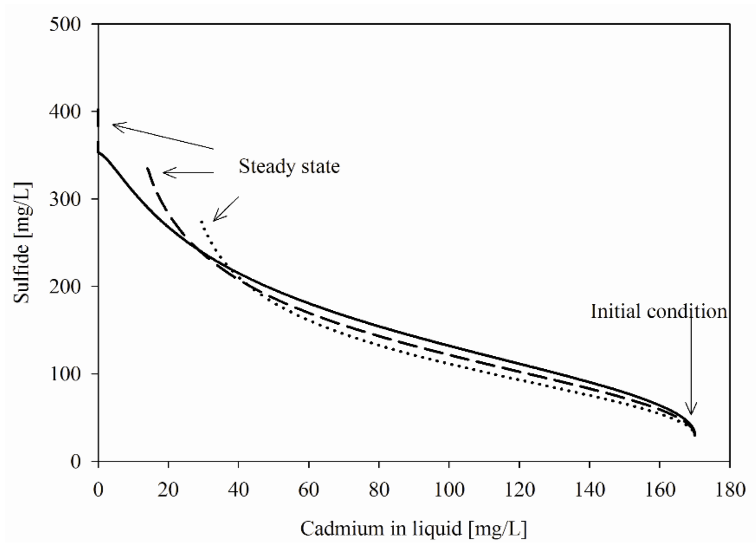


Figure 1. Open-loop phase portrait under different dilution rate values. D = 0.015 1/h (▬▬), D = 0.03 1/h (▬ ▬), and D = 0.045 1/h (•••).

From the above mentioned conditions, the following results are obtained. Figure 1 shows an open-loop phase portrait considering the sulfide, cadmium and biofilm concentrations under different dilution rate values, this figure allows to determinate the adequate values of the considered concentration from the specific dilution rate value, in order to select the corresponding set point for the sulfide concentration which is able to diminish the cadmium concentration to the required value concentration, therefore the sulfide concentration set point is selected as 400 mg/l, which lead a cadmium concentration value of 0.05 mg/L which is an satisfactory level considering the environmental laws.

Figure 2, is related with the performance of the proposed uncertainty estimator, which infers the unknown kinetic term, as can be seen an satisfactory performance is reached, such that the uncertainty observer trajectory reach the named real trajectory, during the open-loop and closed-loop operations periods.

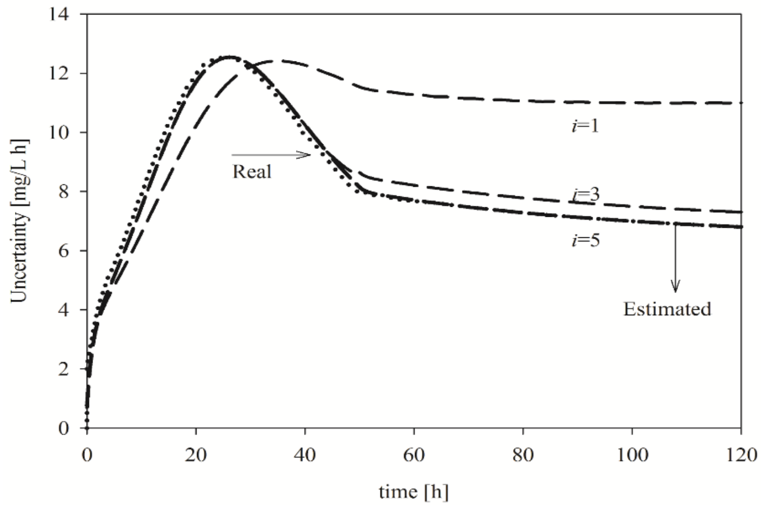


Figure 2. Closed-loop behavior of the uncertainty observer.

Figure 3 shows the performance of the controlled concentration, under the action of the PI controller and the proposed methodology, here is observed the better performance of the proposed controller which leads the sulfide concentration to the required set-point in a short setting time (around 10 hours) without over shoots, in comparison with the PI controller which needs 70 hours to reach the corresponding set-point, presenting, besides, a large overshoot of 200 mg/L before to slow down at the set point.

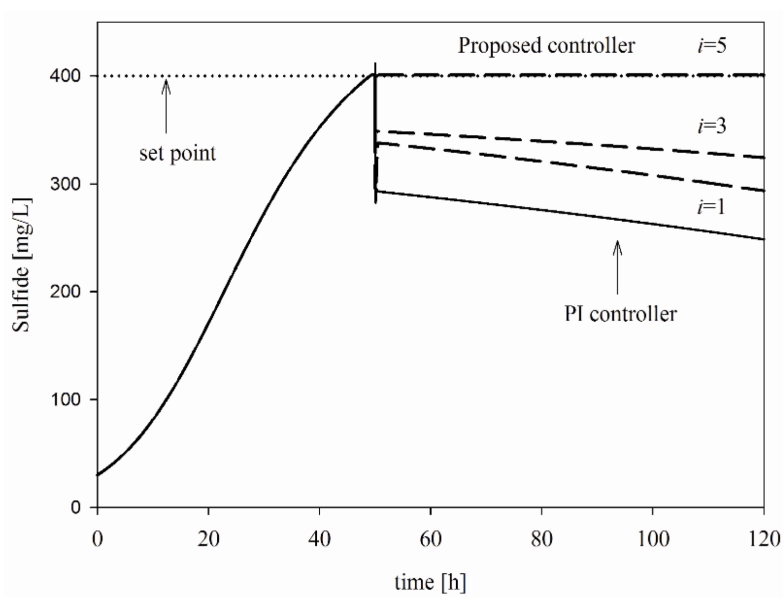


Figure 3. Closed-loop dynamic of the control output.

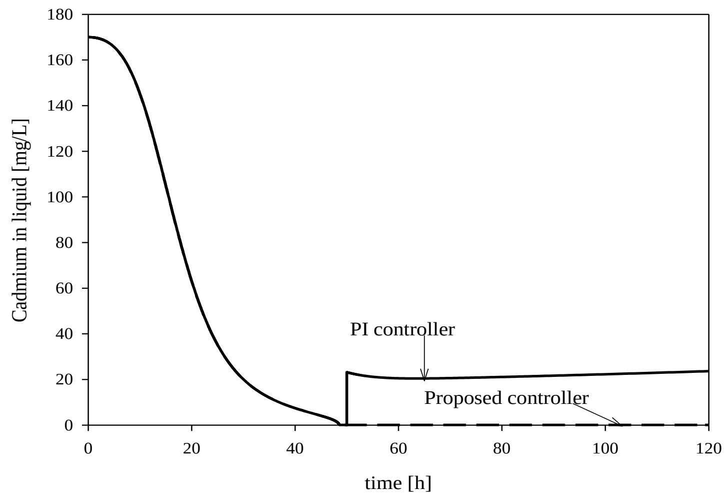


Figure 4. Closed-loop behavior of the Cadmium concentration.

Figure 4 shows the response of the cadmium concentration, under the closed-loop dynamic of the sulfide concentration, note that the cadmium is under the environmental required cadmium concentrations (less than 0.05 mg/l), carrying out with the process objective.

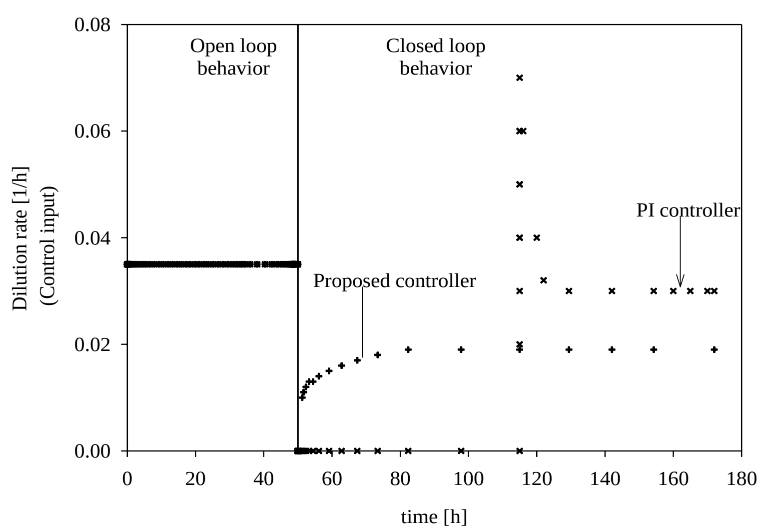


Figure 5. Control effort dynamics.

On other hand, Figure 5 contains the performance of the control effort for both analyzed controllers, as it is observed both controllers act closing the input flow, this lead to discontinuous operation, such that the bioreactor is operating under batch regimen, losing the benefits of the continuous operation; however the proposed controller actives the input flow quickly reaching satisfactory levels (around 0.02 $h^{-1}$) without great effort, in comparison with the PI controller which acts closing the input flow for 65 hours and activating it with an large overshoot reaching to dilution rate value around 0.07 $h^{-1}$.

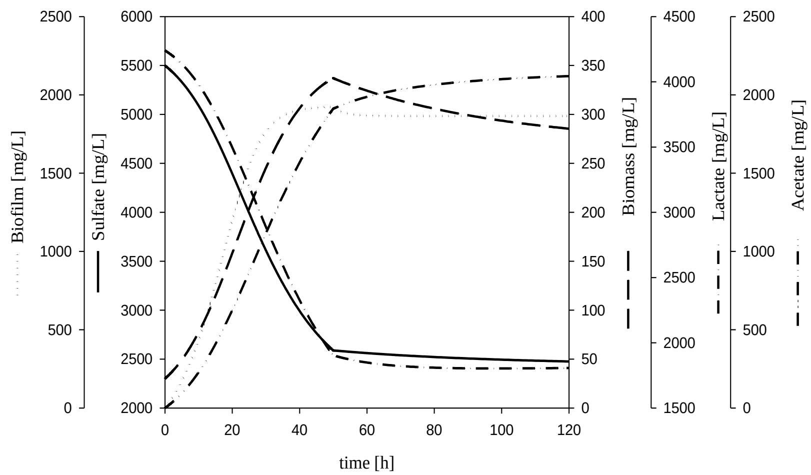


Figure 6. Closed-loop behavior of the uncontrolled concentrations.

Figure 6 contains the performance of the uncontrolled concentrations (sulfate, biomass, lactate, acetate and biofilm) under the action of the proposed control law, notice that the dynamic performance is stable, such that the dynamic trajectories remain with a monotone decreasing behavior until to reach the corresponding closed-loop steady-state.

## Conclusion

Due to uncertainties existed in the applications in biotechnological systems which seriously influence the control performance, an uncertainty observer based control for regulation of cadmium concentration in a wastewater bioremediation system using SRB was applied considering an Input-Output (I/O) linearizing controller in order to design a robust methodology against model uncertainties and input disturbances. It was proved that the closed-loop system is regular, and asymptotically stable via simulation analysis to illustrate the main results. The results obtained here can help to showcase the potential of state and kinetic estimation applied to biological systems and its impact into the development of more robust control systems for such applications.

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